Multiproduct Pricing: Theory and Evidence From Large Retailers*

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Abstract

We study a unique dataset with comprehensive coverage of daily prices in large multiproduct retailers in Israel. Retail stores synchronize price changes around occasional "peak" days when they reprice around 10% of their products. To assess aggregate implications of partial price synchronization, we develop a new model in which multiproduct firms face economies of scope in price adjustment, and synchronization is endogenous. Synchronization of price changes attenuates the average price response to monetary shocks, but only high degrees of synchronization can substantially strengthen the real effects of monetary policy shocks. Our calibrated model generates real effects similar in magnitude to those in Golosov and Lucas (2007).

JEL classification codes: D21, D22, E31, E52, L11.

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1 Introduction

The literature on the effects of monetary policy on the economy has relied extensively on the role of sticky prices in the transmission mechanism. In standard models, firms sell only one product, which means that they face an *inter-temporal* pricing decision. For example, a firm that faces unexpectedly high demand can raise its price right away and incur the cost of price adjustment, or it can delay adjustment and face the rising cost of accommodating extra demand. Most retail firms, however, sell hundreds or thousands of products and, therefore, have an additional *intra-temporal* option to change those prices that are worth adjusting. In this paper, we explore unique data to study how large retailers trade off these inter- and intra-temporal pricing margins. These data inform a new sticky price model that helps us understand how multiproduct pricing influences the real effects of monetary policy shocks.

We examine price data for large multiproduct food retailers in Israel. Unlike most available data on micro prices, the dataset provides comprehensive detail along both dimensions of a firm's price adjustment: time and products. A disclosure law enacted in 2014 requires large retailers in Israel to post on their internet sites daily price information for all products sold in their stores. The data used in this paper contain information for stores representing all large retail chains, 25 in total, from May 20, 2015, until October 4, 2019. For 71 brick-and-mortar stores in our sample we observe "base," or regular, prices for almost all products sold in each store, averaging 7,217 per store on a given day. In all, the dataset contains 506.1 million daily observations.

We exploit comprehensive product and time coverage in our data to document the synchronization of price adjustments across products in a store.[1] Figure 1 depicts the average daily fraction of price adjustments for four selected stores from different chains. It is apparent that stores do a majority of their regular price changes during occasional "peak" days. For example, when we define peaks as the subset of the most active days that jointly account for one-half of all price changes in a store over the entire sample, we find that they occur once every three weeks, on average. On a peak day, a store reprices about 10% of its products, 20 times the number of price adjustments on an average off-peak day. This behavior is not driven by weekly or monthly seasonal effects, Jewish holidays, differences between online and

[1] An extensive empirical literature has studied within-store synchronization of price changes. It typically finds that synchronization of price changes within stores is higher than across stores: Lach and Tsiddon (1996), Midrigan (2011), Bhattarai and Schoenle (2014), Dedola, Kristoffersenz, and Züllig (2019). Levy et al. (1997), Dutta et al. (1999) found that retail prices for similar products tend to be more synchronized. Goldberg and Hellerstein (2009), Bhattarai and Schoenle (2014) use producer price data to document that firms selling more products adjust prices more frequently. Cavallo (2017), Gorodnichenko and Talavera (2017), Gorodnichenko, Sheremirov, and Talavera (2018) find little difference in synchronization of price changes in online and offline stores. DellaVigna and Gentzkow (2019), Hitsch, Hortaçsu, and Lin (2019) provide evidence of uniform pricing by stores within the same retail chain.

offline prices, or retail discounting, suggesting that price synchronization is driven by store-specific fundamentals. Hence, this paper's empirical contribution is to document partial price synchronization in a typical retail store which stems from occasional but regular peaks in stores' repricing activity. We also show that time-averaging can significantly lower measured degree of synchronization, which underscores the importance of using high-frequency price data.

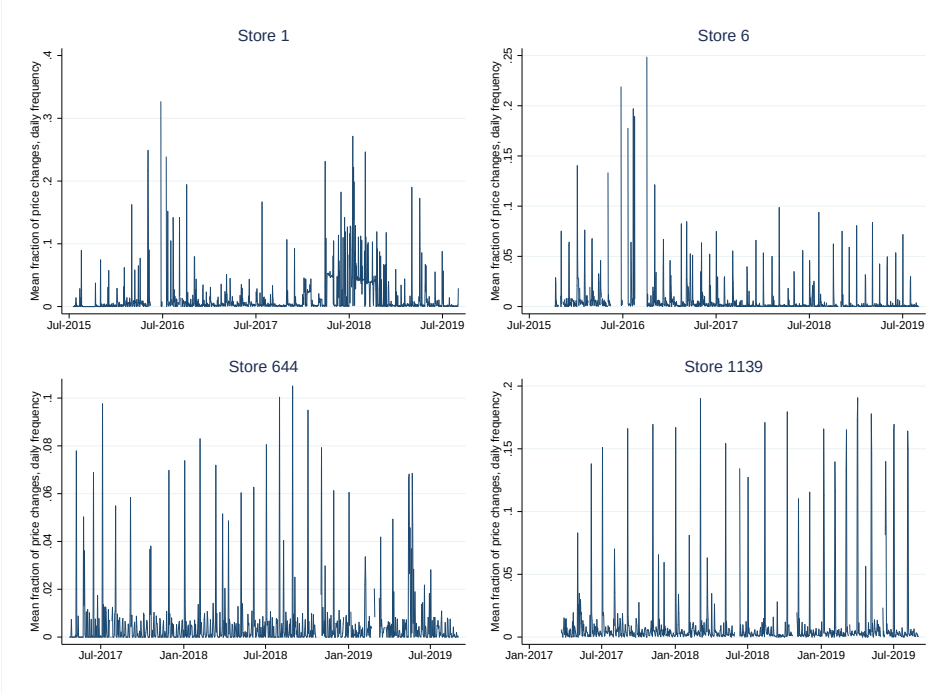


Figure 1: Daily fraction of prices changes, selected stores.
Note: Each plot provides the daily fraction of price changes for one of the four selected (brick-and-mortar) stores in our dataset.

Partial synchronization of price changes highlighted in Figure 1 is incompatible with current sticky price models. These standard pricing theories generally feature either independently priced goods, which generates a staggered pattern of price changes (Golosov and Lucas, 2007), or models with jointly priced products that produce perfectly synchronized price adjustments (Midrigan, 2011; Alvarez and Lippi, 2014). This paper's theoretical contribution is to embed partial synchronization of price changes in an otherwise standard sticky price model and analyze the relationship between the degree of synchronization and the real effects of monetary policy.

To this end, we develop a continuous-time model of multiproduct firms facing economies of scope in price adjustment (Sheshinski and Weiss, 1992). In the model, each firm sells a large number of differentiated goods and faces two types of costs when changing prices: a fixed cost K when at least one price adjustment is made, and an additional cost c for each individual

price change. This general framework gives rise to an endogenous degree of synchronization of price changes that depends on the combination of price-adjustment technology parameters c and K, and it nests two polar cases of independently priced products ($K = 0$) and maximal economies of scope in price adjustment ($c = 0$). The common cost $K > 0$ gives rise to intervals with no repricing activity. The individual menu cost c implies that only prices far enough from their optimal levels are adjusted. So, the multiproduct firm in our model adjusts prices only infrequently, and when it does, it adjusts the prices of a substantial number of products at the same time, but not all of them. It thus generates the key features of partial synchronization pattern observed in the data.

We then study the responses of the aggregate price level and real output to unanticipated nominal aggregate demand shocks. These responses are influenced by the selection effect characteristic of menu-cost models (Golosov and Lucas, 2007). In a model where single-product firms change prices independently from each other, the shock initially triggers adjustment of prices that are farther away from the optimal, amplifying the response of the aggregate price level.[2] In our model, the selection effect is influenced by synchronization of price adjustments. If there is a large degree of synchronization (i.e., many prices are adjusted at the same time) firms have lesser margin for selecting those prices that are further away from their optimal values, weakening the selection effect.

We analyze the degree of monetary non-neutrality in the model calibrated to match the synchronization of price changes in the data, measured using the Fisher and Konieczny (2000) index, as well as the daily fraction and average absolute size of regular price changes. Results indicate that even small deviations from full synchronization can significantly reduce the persistence of real effects of nominal demand shocks. For the degree of synchronization observed in the data, our model generates responses very close to the Golosov and Lucas (2007) model. Although firms in the partial synchronization model do not change all prices at the same time, they change those prices that are farther from the optimal, triggering larger adjustments shortly after the shock. Hence, the selection effect plays a key role in the partial synchronization model, engineering a faster response of the aggregate price level and thus attenuating the real effects of the monetary shock.

Our numerical simulations indicate that monetary non-neutrality is proportional to the kurtosis of price changes for a given frequency of price changes, suggesting that the sufficient statistic result in Alvarez, Le Bihan, and Lippi (2016) also holds in a partial synchronization

[2] Golosov and Lucas (2007) study the selection effect in the recent generation of applied general equilibrium menu-cost models. Earlier theoretical contributions include Caplin and Spulber (1987), Danziger (1999), Caballero and Engel (2007), among others. Carvalho and Kryvtsov (2021) propose a simple, model-free way to measure price selection and its impact on inflation.

setting.[3] In our model, the kurtosis of price changes increases monotonically with the degree of synchronization. Lower synchronization implies a stronger selection effect that, in turn, reduces the kurtosis of price changes relative to the kurtosis of the underlying shocks. Thus, we extend the results in Midrigan (2011) and Alvarez and Lippi (2014) by showing how the *degree* of synchronization affects monetary non-neutrality in a multiproduct setting.

To sharpen the mapping between theory and data, we also study an extension of the Alvarez and Lippi (2014) model in which price adjustments are fully synchronized for subsets of firm's products ("aisles"), but are independent across aisles. This "aisle model" can generate partial synchronization because price adjustments of entire aisles of products produce peaks in the firm-level daily fraction of price changes. Although the calibrated aisle model can match the degree of partial synchronization observed in the data, its aggregate implications remain very close to those of the full-synchronization Alvarez and Lippi (2014) model. Unlike the partial synchronization model, however, the aisle model fails to generate a bimodal distribution of price changes, and it predicts a much smaller dispersion of price spell durations than in the data. These results underscore the importance of including both common and individual menu costs in the price-adjustment technology.

The theoretical literature has emphasized the fundamental link between within-firm price synchronization and macroeconomic effects of monetary shocks. Midrigan (2011) develops a model with two-product firms facing maximal economies of scope in price adjustment and fat-tailed cost shocks. He shows that when the model is calibrated to key moments in micro data, it generates real effects of monetary policy that are five times greater than those effects in the single-firm model in Golosov and Lucas (2007). Bhattarai and Schoenle (2014) demonstrate that models with economies of scope in price adjustment can account for within-firm price synchronization observed for U.S. producers. Alvarez and Lippi (2014) expand Midrigan (2011)'s maximal economies of scope to an arbitrary number of products for the case with Gaussian shocks. They establish analytically that both the size of the output response and its duration increase with the number of products, converging to the response of Taylor (1980) staggered pricing model when the number of products gets large. Hence, both Midrigan (2011) and Alvarez and Lippi (2014) find that multiproduct pricing greatly amplifies the real effect of monetary shocks. This paper demonstrates that full synchronization of price changes in their models is crucial for that conclusion.

The remaining sections of the paper are organized as follows. Section 2 provides new evidence from price data for large food retailers in Israel. In Section 3, we present a multiproduct pricing model with endogenous synchronization of price changes, characterize firms'

[3]The class of models studied in Alvarez, Le Bihan, and Lippi (2016) and Alvarez, Lippi, and Oskolkov (2022) includes the case with a continuum of products but only full price synchronization.

decision functions, and calibrate the model. Section 4 studies analytically and numerically impulse responses to a monetary shock in the model with partial synchronization of price changes. Section 5 studies the alternative aisle model. The last section concludes.

2 Evidence from retail stores in Israel

In this section, we describe the novel dataset from large retail stores in Israel. Building on the dataset's detailed coverage of daily prices for thousands of products in each store, we analyze the degree to which each store synchronizes price changes across different products on a given day. We document a significant degree of partial price synchronization in a typical retail store due to recurrent spikes of the fraction of price adjustments. This synchronization pattern is not due to price discounts, holidays or calendar fixed effects. We show that time averaging can significantly wash out daily variation of price changes. We also provide improved measures of salient pricing moments.

2.1 The Israeli retail data

Large food retailers operating in Israel are required by law to publish online daily price information for all products sold in their stores.[4] The Bank of Israel scrapes, cleans, and consolidates this information on a daily basis. The historical data include information for all products sold by large retailers—a total of 25 retail chains and around 1,700 stores, which account for most of the volume of food retail sales from May 20, 2015, until October 4, 2019. Because the disclosure law applies only to large retailers, the dataset does not include small food retailers, pharmacy retailers, and mom-and-pop stores. Most of the products sold by these retailers are food products, although they also sell non-food products, such as personal care items, cleaning supplies, and small electric appliances. We exclude store-specific products that do not have a general 13-digit barcode (e.g., fruits, vegetables, bakery goods).

To manage computational constraints, for each retailer, we analyze stores in the top 5% by the number of observations. The remaining dataset contains 506.1 million daily observations for 71 brick-and-mortar stores from 25 retail chains. For each store, we have information

[4]Following the "Social Protest" in Israel and the recommendations of the public committee that was formed in response to this social movement, a "Promotion of Competition in the Food Industry Law" was passed by the Israeli Parliament in 2014. The law applies to large food retailers, with annual sales exceeding NIS 250 million (about USD 70 million). According to 2017 Household Income and Expenditure Survey (Table 38 in Israeli Central Bureau of Statistics, 2019), about 60% of food are purchased in chain stores, 15% in groceries, 4% in open market, and the rest in specialty stores.

about "base," or regular, prices for individual products on a daily basis.[5] For four retailers we also have information about their online prices, which we treat as four additional online stores.

A combination of two features of this dataset makes it particularly useful for documenting pricing behavior of large retailers: extensive coverage of products in each store and high frequency of product-specific price observations over time. The number of products sold in a store is large: on average 7,217 products are sold on a given day, with 1,311 (31,847) products in the smallest (largest) store in the dataset. During the sample period, inflation in Israel fluctuated roughly around zero. On a given day, a store changes 0.87% of its prices, with about an even split between price increases and decreases. Each change is quite large, around 20% in absolute value (Table 1, Panel A).

	Sample	Mean number of products	Mean fraction of price changes	Mean absolute size of price changes	Synchronization of price changes
		(1)	(2)	(3)	(4)
A	**Daily obs**	**7,217**	**0.87%**	**20.8%**	**0.259**
B	Peak days	8,341	9.89%	20.6%	0.174
	Off-peak days	7,159	0.48%	20.8%	0.130
C	Pooled, by chain	23,664	0.89%	20.8%	0.235
	Pooled, all stores	301,496	0.99%	20.6%	0.206
D	Weekly obs	8,170	4.28%	20.2%	0.241
	Monthly obs	9,605	11.41%	20.0%	0.201

Table 1: Summary statistics for price adjustments.

Note: The table provides weighted means for statistics in columns (1)–(4) across stores. Panel A provides store-level statistics using daily data (baseline). Panel B provides statistics for peak and off-peak days in each store. Peaks are the days with the highest number of price changes that together account for 50% of all price changes in the store, and "off-peaks" are the remaining days. In Panel C observations are pooled across stores in a chain or across all stores. Panel D provides statistics for observations at weekly and monthly frequencies. Weights are the average number of products in a store per day. Synchronization of price changes in column (4) is measured by Fisher and Konieczny (2000) index defined in Section 2.2.

2.2 Synchronization of price changes

The most striking pattern of daily price changes is evident in Figure 1, which plots daily fraction of price changes in the dataset for four selected brick-and-mortar stores from different chains. It shows that stores occasionally reprice a bulk of their products. As a baseline, we define the peaks in price-adjustment activity for a given store as the set of days with the highest number of price changes that together account for half of price changes in all days for

[5] We report most empirical results for regular prices, and in Appendix A.1 we review results for final prices, which incorporate various types of price discounts.

that store. Peaks are present in all stores in the sample. In a given store, only 5.3% of all days are peaks, i.e., one peak every 19 days on average. Table 1 (Panel B) shows the breakdown of frequencies of price changes for peaks and the remaining days (off-peaks). On a peak day a store reprices about 10% of all products, 20 times the number of price adjustments on an average off-peak day.

To quantify the degree of synchronization of price changes in the data, we use the Fisher and Konieczny (2000) index:

$$FK_s \equiv \sqrt{\frac{\frac{1}{N_s}\sum_t \left(Fr_{s,t} - \overline{Fr_s}\right)^2}{\overline{Fr_s} \cdot \left(1 - \overline{Fr_s}\right)}},$$

where FK_s is the index for store s, $Fr_{s,t}$ is the fraction of price changes in store s period t, $\overline{Fr_s}$ is its mean over t, and N_s is the number of price changes in store s. The FK index is the (square root of the) ratio of the variance of the observed fraction of price changes $Fr_{s,t}$ to $\overline{Fr_s} \cdot \left(1 - \overline{Fr_s}\right)$. So the ratio is 1 when $Fr_{s,t}$ follows a Bernoulli process (prices are perfectly synchronized); and the ratio is zero when $Fr_{s,t}$ has zero variance (prices are perfectly staggered as in Golosov and Lucas (2007) or in most time-dependent models).[6] The weighted mean of the index is 0.259, and it varies considerably across stores, with interquartile range of 0.098. The increase in synchronization of price changes between stores in the 25th and 75th percentiles of the FK index (0.218 versus 0.295) corresponds to a longer average distance between the repricing peaks, by about 19 days.

The literature often relies on other price synchronization statistics, derived from probit or logit regressions (Midrigan, 2011; Bhattarai and Schoenle, 2014; Dedola, Kristoffersenz, and Züllig, 2019). These statistics, however, are not reliable when the distribution of the fraction of price changes places a lot of weight on the extremes, as is the case in our daily data. We demonstrate this result in Appendix A.2, where we provide the estimated probit regression coefficients.[7] By contrast, the FK index captures peaks quite well. Its values for only peak days or only off-peak days (0.174 and 0.130) are lower than for all days together (0.259) reflecting 20-fold difference in the fraction of price changes on peak and off-peak days.

Our evidence suggests that price synchronization is store-specific. We do not find significant differences between synchronization of regular price increases and decreases, or between

[6] See also Dias et al. (2005) for the properties of this index and a statistical test for staggering of price-setting.

[7] For robustness, we analyze yet another synchronization measure that is well-behaved when the fraction of adjustments has peaks. This measure is akin to the Gini inequality index. It is based on the Lorenz curve that maps cumulative fraction of price changes against the percentile of days according to the number of price changes in a day, ranked in ascending order. We demonstrate in Appendix A.2 that FK and Gini synchronization measures are qualitatively in line with each other across different cuts of the data.

regular and final prices (Appendix A.1). Calendar events—holidays and week/month fixed effects—are not important factors for peak adjustments (Appendix A.3). Furthermore, re-pricing peaks are not product-specific: prices of most products in a store adjust on both peak and off-peak days, and peak days mix adjustments of products across the store (Appendix A.4). Finally, partial synchronization for non-food products is similar to food products, suggesting our findings apply to a broader set of consumer products, not just food.[8]

2.3 Improved measurement of price changes

It is well-known that time-aggregation of observations to weekly or monthly frequency may lead to inaccurate measurement of salient pricing moments. For example, Alvarez, Le Bihan, and Lippi (2016) explain how heterogeneity (across products and stores) and measurement error create challenges for measuring kurtosis—the moment associated with their sufficient statistic for the degree of monetary non-neutrality in menu cost models. Cavallo (2018) shows that measurement error introduced by time averaging can lead to underestimation of the average magnitude of price changes. Therefore, high-frequency price observations and comprehensive in-store product coverage in our data are helpful for providing more accurate measures of key pricing moments for large retailers.

Table 1, Panel D shows that time-aggregation of observations to weekly or monthly frequency downplays synchronization of price changes. Table 2 summarizes four statistics for price changes in our data (as in Alvarez and Lippi, 2014): the ratio of the standard deviation of price changes to the mean absolute price change, kurtosis of price changes, and the share of price changes that, in absolute value, are smaller than half (quarter) of the mean absolute price change. To control for heterogeneity of price changes across stores, we construct "standardized" log price changes for product i, store s, and time t, z_{ist}, by subtracting from log price change Δp_{ist} the mean μ_s of non-zero log price changes and dividing by their standard deviation σ_s at a store level: $z_{ist} \equiv \frac{\Delta p_{ist} - \mu_s}{\sigma_s}\Big|_{p_{ist} \neq 0}$. We also remove "outliers" with absolute log price changes in the top 0.5 percentile.

[8] In Appendices A.4 and A.5 we show that synchronization of price changes across stores and across products in a store is in line with findings in the previous literature. We observe that synchronization within stores is higher than across stores (Lach and Tsiddon, 1996; Midrigan, 2011; Bhattarai and Schoenle, 2014; Dedola, Kristoffersenz, and Züllig, 2019). Pooling observations across all stores reduces FK index to 0.206 (Table 1 Panel C). But pooling across stores of the same retail chain only slightly lowers synchronization, suggesting that retailers actively coordinate price changes across their stores (DellaVigna and Gentzkow, 2019; Hitsch, Hortaçsu, and Lin, 2019). Price changes in larger stores tend to be more synchronized than in smaller stores (Goldberg and Hellerstein, 2009; Bhattarai and Schoenle, 2014; Dedola, Kristoffersenz, and Züllig, 2019). Price changes are more synchronized for similar products and for products with more flexible prices (Levy et al., 1997; Dutta et al., 1999). The frequency and size of price changes are very similar for online and offline prices, but online prices are more synchronized (Cavallo, 2017; Gorodnichenko and Talavera, 2017; Gorodnichenko, Sheremirov, and Talavera, 2018).

Moments	Daily	Weekly	Monthly
std($\|z_{ist}\|$) / mean($\|z_{ist}\|$)	0.68	0.67	0.69
kurtosis(z_{ist})	3.5	3.3	3.4
fraction $\|z_{ist}\| < 0.5$*mean($\|z_{ist}\|$)	0.14	0.27	0.28
fraction $\|z_{ist}\| < 0.25$*mean($\|z_{ist}\|$)	0.06	0.10	0.11

Table 2: Standardized regular price changes.

Note: The table reports statistics for standardized log price changes z_{ist} at daily, weekly and monthly observation frequencies. Weekly (monthly) price changes are cumulative sums of daily price changes during the week (month).

Table 2 shows that time aggregation from daily to weekly or to monthly frequency artificially doubles the mass of small price changes. Kurtosis and standard deviation of price changes are almost unaffected, indicating that time aggregation also removes some larger price changes. We estimate kurtosis at around 3.5, which is slightly lower than 4 documented in Alvarez, Le Bihan, and Lippi (2016) using Dominick's weekly scanner data and the range between 4 and 5.5 estimated in Cavallo (2018) using daily online prices for 31 countries, including non-food prices. In Appendix A.5, we show that results are not driven by heterogeneity across products within a store. Furthermore, because we analyze posted prices, there is no imputation error common in analyses of scanner data (Eichenbaum et al., 2014). Indeed, when we obtain price-setting statistics from IRI Symphony weekly scanner data for the United States, we find higher share of small price changes and higher kurtosis, which could be partly explained by imputation errors.

3 A model with multiproduct firms and partial synchronization

In this section, we introduce a multiproduct price-setting model that is capable of generating the synchronization patterns we observe in the data, i.e., occasional peak days on which a substantial fraction of prices within a store is reset. This *partial synchronization* model features two key ingredients: a fixed cost required to make any number of price adjustments, and an additional menu cost that is paid for each individual price change. We then propose a solution method based on a recursive formulation of the problem. Finally, we calibrate the model to match the FK synchronization index, as well as the frequency and size of price changes in the data, and compare its predictions to those of two prominent models—the Alvarez-Lippi-Midrigan and Golosov-Lucas models.

3.1 Model overview

Each firm sells a continuum of differentiated goods, with total mass normalized to one. We also refer to these goods as varieties or products. Given the large number of products per store in our data, the assumption that firms in our model sell a continuum of products is suitable for our purposes. Each variety is indexed by $i \in [0, 1]$ and has a frictionless optimal price $p^*_{i,t}$, where t indexes time, which flows continuously. All prices are in log units. The frictionless optimal price $p^*_{i,t}$ is the profit-maximizing price for good i in the absence of frictions. In the presence of adjustment costs, good i's price at instant t, $p_{i,t}$, may differ from the frictionless price, leading to a profit loss.[9] We can write the profit loss at time t using a second-order approximation, as follows:

$$L_t = \int_0^1 (p_{i,t} - p^*_{i,t})^2 \, di \, . \tag{1}$$

Intuitively, this expression is the sum of profit losses associated with sub-optimal prices across all products. Firms discount future costs at a rate ρ. We assume that each product i's frictionless optimal price follows a Brownian motion:

$$dp^*_{i,t} = -\sigma dW_{i,t} \, ,$$

where $W_{i,t}$ is a variety-specific standard Brownian motion assumed to be independent across goods, and σ is a parameter that captures its volatility. To maintain tractability, we assume the optimal price of a given product does not depend on the prices of other varieties. This assumption eliminates feedback effects from firms' pricing decisions to desired prices and greatly simplifies the solution for the aggregate price response to a common shock, provided in Section 4.

It is simpler to express the firm's problem in terms of *price discrepancies,* or *price gaps*, which are defined as $x_{i,t} = p_{i,t} - p^*_{i,t}$. The law of motion for $p^*_{i,t}$ implies that price discrepancies, in the absence of price adjustments, are also Brownian motions of the form

$$dx_{i,t} = \sigma dW_{i,t} \, .$$

It is convenient to state the loss function in terms of the distribution of these price discrepancies. Let $g_t(x)$ be the probability density function (p.d.f) that describes the distribution

[9] Appendix B.1 provides the microeconomic foundations for the underlying general equilibrium model, based on Alvarez and Lippi (2014) and Bonomo et al. (2021).

of price gaps.[10] We can express the loss term (1) as

$$L_t = \int_{-\infty}^{+\infty} x^2 g_t(x)\, dx\,. \tag{2}$$

This is essentially a change of variables in equation (1). Instead of summing the losses associated with each product, we now sum the loss x^2 associated with each price gap $x = p - p^*$, multiplied by the number of times (or density, more precisely) such a gap occurs $g_t(x)$. The evolution of $g_t(x)$, given an initial distribution $g_0(x)$, is determined by a Kolmogorov forward equation (KFE):

$$\frac{\partial g_t}{\partial t}(x) = \frac{\sigma^2}{2}\frac{\partial^2 g_t}{\partial x^2}(x)\,. \tag{3}$$

The last component of the firm's problem are the pricing frictions. We assume that firms face menu costs of two different kinds. First, there is a fixed cost K that firms must incur to make any number of price adjustments. Second, there is an additional menu cost c that applies to the fraction of prices being adjusted. Therefore, in order to adjust prices of a fraction m of its products, a firm must pay $K + cm$.

These pricing frictions give rise to optimal policies that have two important features. First, a positive common cost K generates common inaction: there will be time intervals in which the firm does not adjust any price. Second, individual menu cost c implies that when a firm adjusts its prices, it does not adjust all of them. Intuitively, there are always products with arbitrarily small price discrepancies (in absolute value) for which it is not optimal to pay the unit menu cost c and set the discrepancy to zero.

The relevant state variable in the model is the distribution of price gaps. Since each firm sells a continuum of products, in steady state this distribution evolves deterministically, according to (3). Given an initial distribution $g_0(x)$, the optimal policy consists of sequences of deterministic adjustment dates $\{T_k\}_{k=1}^{\infty}$ and thresholds $\{\bar{x}_k\}_{k=1}^{\infty}$ such that at instant $t = T_k$ the firm adjusts all prices that have price gaps x larger, in absolute terms, than $\bar{x}_k$, that is $|x| \geq \bar{x}_k$. Since there is no trend inflation in the model, every price change optimally sets the discrepancy to zero.[11] Consequently, the distribution of price gaps will feature a mass at

[10] The distribution of price discrepancies may have atoms on adjustment dates. For simplicity, our notation abstracts from this possibility.

[11] In the presence of trend inflation, the optimal sequence of thresholds has to be split into sequences of upper thresholds $\{\bar{x}_k\}_{k=1}^{\infty}$, lower thresholds $\{\underline{x}_k\}_{k=1}^{\infty}$, and targets $\{x_k^*\}_{k=1}^{\infty}$ such that a price is adjusted at date T_k only if the corresponding discrepancy x satisfies either $x \geq \bar{x}_k$ or $x \leq \underline{x}_k$. The discrepancy is set to x_k^*, which is not necessarily zero. In other words, a positive (negative) inflation trend leads to expectations that price gaps will fall (rise). In this case, adjusting firms will optimally reset prices to a level above (below) their frictionless optima.

$x = 0$ on adjustment dates. These masses are, however, instantly dissolved by the diffusive nature of the Brownian motion.

To see why the optimal policy takes the form of thresholds $\{\bar{x}_k\}_{k=1}^{\infty}$ just described, notice that once the common cost K is paid, adjusting the price of a given good does not affect other goods' price gaps and the expected flow of future costs that arise from them. Therefore, after paying the common cost, the firm will decide independently to adjust the price of a product whenever its price gap is high enough to make the benefit of adjustment higher than c. Since the benefit of adjusting a product's price is increasing in its price gap, there is a minimum level, $\bar{x}_k$, such that it is optimal to adjust all products with gaps larger than $\bar{x}_k$.

3.2 Recursive formulation

The main difficulty in solving the partial synchronization model is that the relevant state variable in the dynamic optimization problem is the entire distribution of price discrepancies. Alvarez and Lippi (2014) show that, in the perfect synchronization case ($c = 0$), there is no need to keep track of the whole distribution. In their model, all relevant information for the firm can be summarized by a one-dimensional object, namely the loss term (1). This dimensionality reduction does not apply in our framework, and we must state the Bellman equation for a value function that takes as input an infinite dimensional object. Before proceeding to the recursive formulation, however, it is convenient to go through two simple mathematical results.

Lemma 1. *Let $\phi(\cdot)$ denote the p.d.f. of a standard normal distribution. Given an initial condition $g_0(x)$, the solution of the KFE (3) is*

$$g_t(x) = \int_{-\infty}^{+\infty} \frac{1}{\sqrt{\sigma^2 t}} \phi\left(\frac{x-y}{\sqrt{\sigma^2 t}}\right) g_0(y)\, dy\,. \tag{4}$$

Proof. See Appendix B.2. □

Lemma 2. *In the absence of price adjustments, the loss term (2) evolves linearly according to*

$$L_t = L_0 + \sigma^2 t\,. \tag{5}$$

Proof. See Appendix B.2. □

Lemma 1 expresses the distribution of price gaps at time t as the convolution of the initial distribution g_0 and a normal p.d.f. This is intuitive: the distribution at time t corresponds to the "sum" of the initial distribution and an independent Gaussian shock, which reflects

the fact that price gaps follow independent stochastic processes with normally distributed increments. Lemma 2 simply reflects the fact that the variance of a Brownian motion increases linearly with time, as it is the sum of independent, equally distributed components.

Now let $V(g)$ denote the value function of a firm, which takes as input the distribution g of price gaps. We state the problem recursively for the case in which g is the distribution of price discrepancies immediately after the payment of the fixed cost K, but *before any price adjustments take place.*[12] The function V then satisfies

$$V(g) = \min_{\bar{x},\tau} \; c\,m(\bar{x}, g) + \int_0^\tau e^{-\rho t}(L_0 + \sigma^2 t)\,dt + e^{-\rho\tau}[K + V(g_\tau)]\,, \tag{6}$$

where the choice variable $\bar{x}$ is the adjustment threshold, and τ is the amount of time the firm decides to wait until the next adjustment date. The function $m(\bar{x}, g)$ gives the mass of reset prices, defined as

$$m(\bar{x}, g) = \int_{|x|\geq\bar{x}} g(x)\,dx\,.$$

L_0 is the instantaneous loss associated with the intermediate distribution $g_0(x)$, which is the distribution of price discrepancies immediately after adjustments are made, given by

$$g_0(x) = g(x)1(|x| < \bar{x}) + m(\bar{x}, g)\delta_0(x)\,,$$

where $1(\cdot)$ is an indicator function and $\delta_0(x)$ is the Dirac function centered at $x = 0$. Since prices with discrepancies larger than $\bar{x}$ are reset, the distribution $g_0(x)$ is simply $g(x)$ with the tails removed and their mass transferred to $x = 0$. Finally, $g_\tau(x)$ is the solution of the KFE (3) at the next adjustment date τ, given the initial condition $g_0(x)$, and is computed using (4).

The interpretation of equation (6) is the following. After paying the fixed cost K, the firm adjusts prices whose gaps lie in the tails of the distribution($|x| > \bar{x}$). This amounts to a mass $m(\bar{x}, g)$ of products, and thus generatesa cost $c\,m(\bar{x}, g)$. After resetting prices, the firm is left with a new distribution of price gaps $g_0(x)$ that generates instantaneous loss L_0. Since the evolution of $g_0(x)$ is deterministic, given by (3), the firm then chooses how long to wait (τ units of time) until the next price-adjustment date, when it pays the fixed cost K and obtains continuation value $V(g_\tau)$. In the meantime, the firm incurs losses that grow linearly over time, as given by (5).

Finally, solving the Bellman equation above gives us optimal policies $\bar{x}(g)$ and $\tau(g)$. We

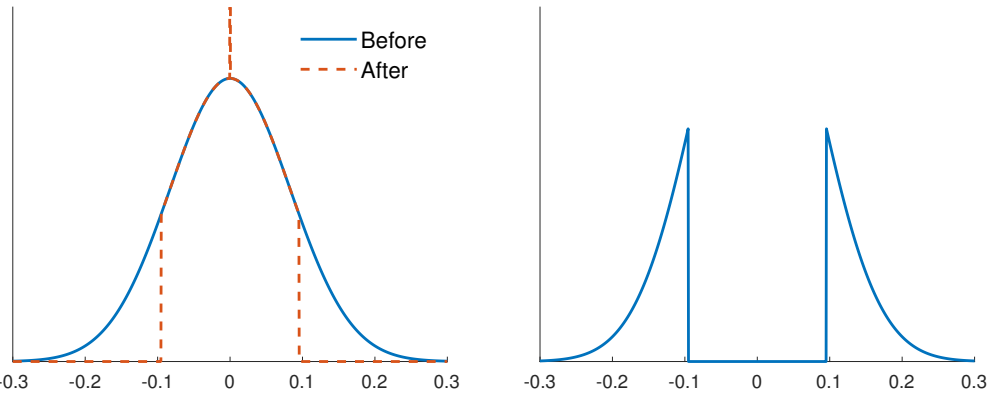
[12]This choice of timing is convenient for the numerical procedure we use to solve the model, which involves using a simple, yet precise, approximation for this distribution, as explained in Appendix B.3.

then define a *steady-state distribution* g^* as a p.d.f., with corresponding optimal policies $\tau^* = \tau(g^*)$ and $\bar{x}^* = \bar{x}(g^*)$, which remains unchanged after the process of resetting prices according to the discrepancy threshold $\bar{x}^*$ and waiting time periods τ^* until the next adjustment date. Therefore, when the system starts from the distribution g^*, the trajectory of the state distribution repeats itself every τ^* periods. Formally, we have:

Definition 1. A *steady-state distribution* is a p.d.f. g^* and optimal policies $\tau^* = \tau(g^*)$ and $\bar{x}^* = \bar{x}(g^*)$, which solves the fixed-point problem:

$$g^*(x) = \int_{-\infty}^{+\infty} \frac{1}{\sqrt{\sigma^2 \tau^*}} \phi\left(\frac{x-y}{\sqrt{\sigma^2 \tau^*}}\right) \left[g^*(y) 1(|y| < \bar{x}^*) + m(\bar{x}, g^*)\delta_0(y)\right] dy\,. \tag{7}$$

This fixed-point problem is easily solvable by creating a discrete grid for possible values of x, since (7) writes g^* as a linear transformation of the term in square brackets. This term, which is itself a linear function of g^*, is the distribution of price gaps immediately after prices are adjusted. If we represent $g^*(\cdot)$ by a vector of the values it attains in the x grid, the problem boils down to finding an eigenvector of a large matrix. Figure 2a shows the steady-state distribution of price gaps g^* and g_0^*, before and after price adjustments respectively, and price changes for illustrative parameter values. The spike at $x = 0$ is the finite grid analog of a Dirac mass, and the price-change distribution corresponds to the tails of the stationary distribution g^* (Figure 2b).



(a) Distribution of price gaps before and after adjustments.

(b) Price-change distribution.

Figure 2: Steady-state and price-change distributions.
Note: Parameter values are $\rho = 0.04$, $\sigma = 0.25$, $K = 0.0001$, $c = 0.001$.

Figure 3 shows the fraction of price adjustments on a daily basis for two different combinations of K and c. Similar to the data, our model generates peaks over time. It is also

interesting to notice how the combination of common and individual costs alters the pattern of peaks. A high common cost K, combined with a low individual cost c is associated with higher but less frequent peaks, as expected. In turn, a low K, high c parametrization generates more frequent but lower peaks.

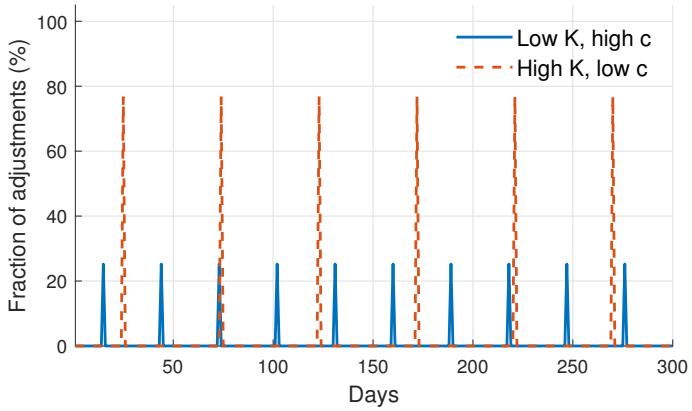


Figure 3: Daily fraction of prices changes for different combinations of menu costs.
Note: Low K, high c: $K = 0.0001$, $c = 0.001$; high K, low c: $K = 0.0005$, $c = 0.0001$. Other parameter values: $\rho = 0.04$, $\sigma = 0.25$.

Our partial synchronization model nests two other cases previously studied in the literature on price setting. Midrigan (2011) and Alvarez and Lippi (2014) study models in which firms are required to pay a single menu cost to reset all prices at once. In those cases, the economies of scope of adjusting prices are maximal and, at any given instant, the share of prices that a given firm resets is either zero or one. In our model, setting $c = 0$ yields the infinite-product limit of the Alvarez-Lippi-Midrigan framework.

The other extreme case is $K = 0$. In this case, there are no economies of scope in adjusting prices, and we can imagine each firm in our model as a continuum of independent firms subject to idiosyncratic shocks, each responsible for adjusting the price of a single good, as in Golosov and Lucas (2007). Firms continuously reset prices that reach certain adjustment thresholds, and the law of large numbers thus guarantees that in any given time interval the share of adjusting prices is constant. Our model therefore flexibly captures pricing behavior ranging from perfect within-firm synchronization of price adjustments to complete lack of synchronization.

3.3 Calibration

To compare predictions of different models, we calibrate not only the partial synchronization model, but also the Golosov-Lucas (GL) and Alvarez-Lippi-Midrigan (ALM) models. Since we set the time discount rate at $\rho = 0.04$, there are three parameters left to be calibrated

in the partial synchronization model: the volatility σ, the fixed cost K, and the unit cost c. We need therefore three moments from the data. As usual in the literature on price setting, we use the frequency of price adjustments and the average absolute size of price changes. Our measure of the frequency of adjustments is the average daily share of prices that a firms adjusts. To match the degree of synchronization of price adjustments by a multiproduct firm, we use the FK synchronization index as our third moment.

The GL and ALM models have two parameters each, since they feature only one menu cost. We therefore drop the FK index when calibrating these models. This is a natural choice since FK statistics are fixed in those models: zero in GL and one in ALM.

Table 3 shows moments for models and data, as well as calibrated parameter values. All models are able to match the average frequency and magnitude of price changes. As discussed previously, however, only the partial synchronization model can match the FK index.[13] As for untargeted moments, all models generate reasonable values for the standard deviation of the distribution of price changes. None of the models employed here succeed in matching the fraction of small adjustments, defined as those smaller (in absolute value) than one-quarter of the average adjustment size. The GL and partial synchronization models are not able to generate any small adjustment, an issue well known in menu-cost models (Midrigan, 2011). In contrast, the ALM model generates too many small price changes. As for the kurtosis of the price-change distribution, only the ALM model is able to generate a value close to the data, as the price-change distribution in this model is always normal, thus implying a kurtosis of 3. In Section 4, we discuss extensions that match these two additional moments.

Figure 4 compares price-change distributions for all three models and data. Since in GL firms continuously adjust prices that reach certain thresholds, the price-change distribution consists simply of two mass points on these limits. On the other hand, in the ALM model, firms only pay the fixed cost K to adjust all prices, even those prices that are close to their profit-maximizing levels. Thus, the price-change distribution features many small price adjustments. The partial synchronization model features more variability in the size of price changes than GL and, contrary to ALM, generates a bimodal distribution of price changes.

Figure 5 shows the daily fraction of adjustments for all models over time. As expected, it is constant for GL case and assumes only the values zero and one in ALM. The partial synchronization model is therefore the one that comes closest to replicating the pattern of peaks seen in the data (Figure 1). The optimal policy for the partial synchronization model

[13] Introducing aggregate shocks to the model could increase synchronization, as prices would respond to a common driver. Time-aggregation or a finite number of products should also affect the FK measure of synchronization. In order to quantify these effects, we simulate a daily panel of 7200 products adding common shocks with 5% annual standard deviation, imposing the same optimal policy $\bar{x}^*, \tau^*$. The resulting increase in FK-synchronization is very modest – from 0.236 to 0.237.

Moment	Data	GL	ALM	Partial sync.
Targeted moments				
Daily fraction of price changes	0.0087	0.0087	0.0087	0.0087
Average $\lvert\Delta p\rvert$	0.208	0.208	0.208	0.208
Fisher-Konieczny index	0.259	0.000	1.000	0.259
Additional moments				
Standard deviation of Δp	0.248	0.208	0.260	0.210
Fraction of small adjustments	0.06	0	0.16	0
Kurtosis of Δp	3.53	1	3	1.09
Calibrated parameter values				
Volatility of the Brownian motion, σ		0.370	0.464	0.373
Common adjustment cost, K		0	0.010	3.70e-05
Individual adjustment cost, c		0.0023	0	0.0020

Table 3: Moments from data and calibrated models.

Note: Values in the data are weighted means across stores in the data, provided in Table 1, row A, columns (2)–(4). Small adjustments refers to standardized daily price adjustments that are smaller (in absolute magnitude) than one-quarter of the average standardized adjustments, provided in Table 2, panel A, last column.

in our calibration consists of adjusting prices with corresponding gaps larger than 17.4% every 8.6 days.

The presence of two types of menu costs is key for matching the pricing patterns observed in the data. The individual cost c, absent in the ALM model, generates bimodality in price changes, as displayed in the data. The fixed cost K generates time intervals without adjustments, leading to synchronization. Additionally, individual costs make small adjustments not optimal, leading to partial synchronization of price changes, as we document in this paper. We now turn to the real output effects of nominal aggregate demand shocks in the partial synchronization model.

4 Real effects of nominal aggregate demand shocks

In this section, we analyze the implications of the partial synchronization model studied above for the real output effects of shifts in aggregate demand, which can be interpreted as monetary shocks. We start by analytically characterizing the price level response in the partial synchronization model, both for individual firms and for the aggregate economy. We then study how monetary non-neutrality varies with the degree of price synchronization. Finally, we document a monotonic relationship between partial synchronization and the kurtosis of

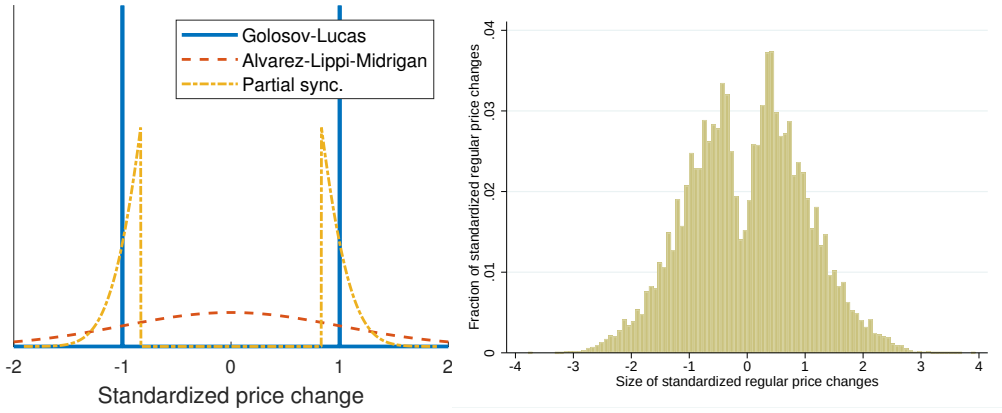


Figure 4: Price-change distributions in models and data.

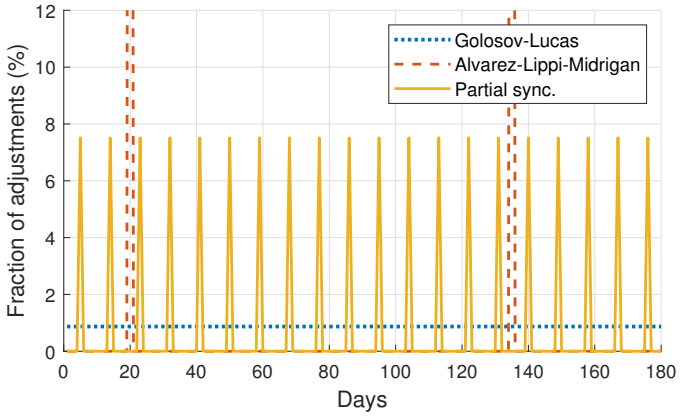


Figure 5: Daily share of adjusted prices for the three models. Spikes in the ALM model feature 100% price adjustments.

the distribution of price changes, which allows our results to be interpreted through the lens of the sufficient statistic presented in Alvarez, Le Bihan, and Lippi (2016).

4.1 Analytical results for the partial synchronization model

In our model, each frictionless optimal price is the sum of a product-specific Brownian motion and an aggregate demand component M_t that has so far been held constant. Let P_t denote the aggregate price level, in logs, which is simply the average price across all firms and products in the model economy. Log real output Y_t is then given by

$$Y_t = M_t - P_t.$$

We now consider the responses of the aggregate price level and real output to a one-time, unanticipated shock ε to M_t at $t = 0$. This shock shifts aggregate demand M_t to $M_t + \varepsilon$ and, as a consequence, the price gap distribution of all firms is also shifted: $g_t(x)$ becomes $g_t(x+\varepsilon)$.

Prior to the occurrence of the aggregate shock, all firms are in steady state, adjusting prices every τ^* periods. We start with a situation in which firms are uniformly distributed according to the time elapsed since the last adjustment date, that is, a constant flow of firms adjust prices over time before $t = 0$. As in the Taylor (1980) model, this is a natural assumption, since it is the only stationary distribution in a steady state without aggregate shocks. [14]

To understand how aggregates respond to such a shock, we must first understand the optimal policy following the shock.[15] Consider a firm that had its last adjustment date at instant $t = -s$, for a given $0 < s < \tau^*$. Recall that the firm's problem is deterministic, since the evolution of the relevant state variable—the distribution of price gaps—is perfectly predictable and can be computed using the KFE (3). Therefore, after the unanticipated shock of size ε is realized at $t = 0$, the optimal policy can be represented by a new deterministic sequence of adjustment dates $\{T_k(\varepsilon)\}_{k=1}^{\infty}$ and thresholds $\{\bar{x}_k(\varepsilon)\}_{k=1}^{\infty}$ that depend only on ε and (implicitly) on s. Also, define Δ_k as the change in the firm's average price in the k-th adjustment episode following the shock, which is a function of the shock size ε and the optimal policy, although we omit this dependence for conciseness. In the absence of changes to aggregate demand ($\varepsilon = 0$), the firm would follow its steady-state policy characterized by zero changes in its average price ($\Delta_k = 0$), and

$$T_1(0) = \tau^* - s,$$

$$T_{k+1}(0) = T_k(0) + \tau^*,$$

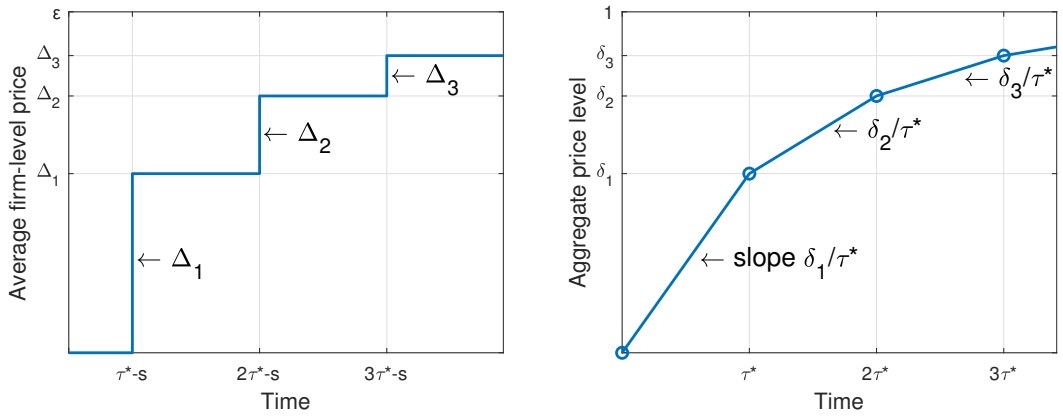
$$\bar{x}_k(0) = \bar{x}^*.$$

To obtain analytical results for the responses of the aggregate price level and output to nominal demand shocks, we focus on the limit as $\varepsilon \to 0$. As ε decreases, the adjustment dates converge to the ones that would arise in steady state, as long as these dates vary continuously with ε. Consequently, the firm's average price will take discrete steps on dates of the form $k\tau^* - s$, as illustrated in the left panel of Figure 6.

Since we assume that firms' adjusting times are uniformly distributed, and that adjusting intervals are τ^*, it follows that at each instant there is a flow $1/\tau^*$of adjusting firms. Moreover, for $t \in (k\tau^*, (k+1)\tau^*)$, the average prices of adjusting firms change by the same amount Δ_k, so the aggregate price level changes at a constant rate Δ_k/τ^*. More precisely, define $P_\varepsilon(t)$ to

[14] Any other distribution would make the inflation response to a monetary shock dependent on the calendar date on which it occurs. For example, if all firms adjust prices simultaneously, a shock that happens shortly after this common adjustment date would feature more monetary non-neutrality than one immediately before it.

[15] Since we employ a second-order approximation to the objective function, there are no general equilibrium feedback effects of an aggregate shock to the firm's optimal policy (Alvarez and Lippi, 2014).



(a) Average price in a given firm. (b) Normalized aggregate price response.

Figure 6: Price response to an aggregate shock of size ε.

be the aggregate price level at instant t following a shock of size ε and let

$$\delta_k = \lim_{\varepsilon \to 0} \frac{\Delta_k}{\varepsilon}.$$

We obtain the following result.

Proposition 1. *The normalized aggregate price response $P_\varepsilon(t)/\varepsilon$ converges to a piece-wise linear function with kinks at positive multiples of τ^* as $\varepsilon \to 0$. Moreover, the slope of the k-th line segment is δ_k/τ^*.*

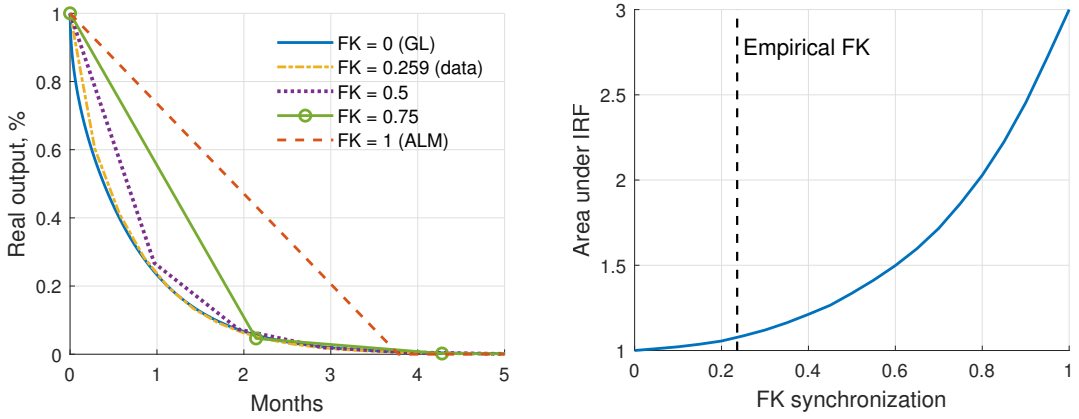
Proof. See Appendix B.2. □

The right panel of Figure 6 shows the limiting response of the aggregate price level to a small positive shock to nominal aggregate demand, as established in Proposition 1. The slope of the first line segment of the impulse response function is quantitatively important since, as we show below, it accounts for almost 40% of the rise of the aggregate price level in the calibrated partial synchronization model.

The algorithm we employ for computing the sequence $\{\delta_k\}_{k=1}^{\infty}$ is straightforward. We simulate the average price of a firm that starts at $t = 0$ with price gap distribution $g^*(x + \varepsilon)$ and follows the pricing policy $T_k = k\tau^*$ and $\bar{x}_k = \bar{x}^*$. As shown in Appendix B.4, the symmetry of the problem allows us to ignore the effects of the shock on pricing policies when computing the first-order response, so we can keep policies at their steady state values. This procedure gives us the sequence $\{\Delta_k\}_{k=0}^{\infty}$, which allows us to compute the full IRF.

4.2 Synchronization and real effects of nominal demand shocks

We now turn to the analysis of the relationship between synchronization of price changes and the magnitude of output response to nominal demand shocks. Figure 7a shows impulse response functions of several models calibrated to match the same frequency of price adjustments and average size of price changes, but different values of the FK synchronization measure. Output decays faster in models with lower synchronization. Since all models are calibrated to match the same frequency of adjustment, the differences in responses must come from the selection effect characteristic of menu cost models.[16] If price changes are very synchronized, firms adjust a large fraction of their prices simultaneously, and there is not much room for selecting those prices. As a consequence, monetary shocks have more persistent real effects.



(a) Output responses to monetary shock.

(b) Area under IRF, normalizing $GL = 1$.

Figure 7: Monetary non-neutrality and synchronization.

The impulse response in the calibrated partial synchronization model is very close to the response in GL, implying much less monetary non-neutrality than in ALM. To help explain this quantitative relationship between synchronization and monetary non-neutrality, Figure 7b shows the area under the output impulse responses as a function of the FK index. This relationship is convex: even a small departure from full synchronization can considerably reduce the degree of monetary non-neutrality. Moderate levels of synchronization, as observed in the data (vertical dashed line), imply weak non-neutrality in the calibrated partial

[16] Golosov and Lucas (2007) study the selection effect in the recent generation of applied general equilibrium menu-cost models. Earlier theoretical contributions include Caplin and Spulber (1987), Danziger (1999), Caballero and Engel (2007), among others. In Appendix B.4 we extend analytically the results in Caballero and Engel (2007) to a multiproduct setting with endogenous price synchronization. We show that the slope of the first segment of the aggregate price impulse response (Figure 6) is proportional to a measure of the selection effect.

synchronization model, quantitatively similar to the non-neutrality obtained in the absence of synchronization.

The initial slope of the output impulse response function is also informative of the degree of selection generated by price synchronization. For a given frequency of price changes, the ALM model delivers the slowest return of output to steady state. On the other extreme, the GL model can be seen as the limit in which the speed of the initial output reversal is maximal.

4.3 The relationship between synchronization, the kurtosis of price changes, and monetary non-neutrality

How do our conclusions relate to the sufficient statistic result derived by Alvarez, Le Bihan, and Lippi (2016) (henceforth, ALL)? ALL show that, in a large class of pricing models, under the assumption that frictionless optimal prices follow Brownian motions, the area under the impulse response function generated by a small shock to nominal aggregate demand is proportional to the kurtosis of the price-change distribution (for a given frequency of price changes).[17] Our partial synchronization model is not included in that class of models. Nevertheless, the ALL result holds numerically for our partial synchronization model as well. Figure 8 shows that as kurtosis of price changes varies across models with the same frequency but different synchronization of price changes (solid blue line), the ratio of kurtosis to the area under the impulse response function remains roughly constant (red dashed line). Importantly, the ALL formula does encompass the perfect synchronization case ($FK = 1$). Therefore, our numerical results not only suggest that the kurtosis-area ratio is constant, but also that it equals the value implied by ALL's sufficient statistic.

Since synchronization and kurtosis of price changes are directly related in our model, and the ratio of kurtosis to the cumulative impulse response appears to be constant, we can interpret monetary non-neutrality in the partial synchronization model through the lens of the ALL sufficient statistic. On one extreme, the GL model features both the smallest possible value for synchronization (zero), and kurtosis (one), as well as the minimal monetary non-neutrality. As we increase the degree of synchronization of price adjustments, the kurtosis of the price-change distribution increases monotonically until it reaches a maximum of 3 for the ALM model, when monetary non-neutrality is also maximal.

[17] Alvarez, Lippi, and Oskolkov (2022) extend the sufficient statistic result to menu cost models with a generalized hazard function. Karadi and Reiff (2019), Dotsey and Wolman (2020) provide examples of models in which the conditions for the ALL result are not satisfied and the ALL sufficient statistic does not hold. In Karadi and Reiff (2019) idiosyncratic shocks are drawn from a mixture of two normal distributions. In Dotsey and Wolman (2020) firms' idiosyncratic productivity processes follow a stationary Markov chain.

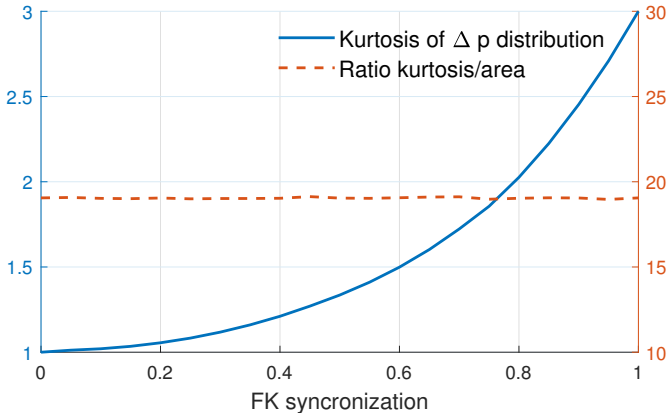


Figure 8: Kurtosis of price-change distribution for various values of the FK synchronization measure.

The kurtosis of the price-change distribution documented in the Israeli data poses a challenge to our model. As can be seen in Table 3, with Gaussian shocks it is not possible to simultaneously match the kurtosis of price changes and the degree of synchronization of adjustments. The degree of synchronization in the data is relatively small, which, as Figure 8 shows, is associated with low kurtosis.

One way of increasing kurtosis in the model is to introduce free Calvo (1983) adjustments, as in ALL and Nakamura and Steinsson (2010). In such a framework, each price is subject to random free adjustment opportunities that arrive at a constant rate and independently across products (see Appendix B.5 for a more detailed exposition). It turns out that this modification of the model does not meaningfully affect our findings. As shown in both Table 3 and Figure 4, the share of small price changes is small in our sample, and therefore introducing free adjustments to match this statistic does not make a substantial difference for the kurtosis of the price-change distribution. Not surprisingly, the output impulse response function to a monetary shock remains almost unchanged in this case.

5 An ALM model with product aisles

In this section, we study a model that generates partial synchronization in price adjustments, but relies on different price-adjustment technology. Consider a multiproduct firm that sells a large number n_{prod} of products. As in the other models, each product has a price gap that, absent adjustments, follows an independent Brownian motion. The main difference is that each product is assigned to one of n_{aisles} equally-sized aisles. Each aisle behaves independently from the others and faces pricing frictions as in ALM, i.e., each aisle requires a single menu cost to adjust the prices of all its products. In other words, the multiproduct

firm sets prices in the same way as n_{aisles} independent multiproduct firms, each facing ALM pricing frictions.[18]

The reason why this model generates partial synchronization is evident: price adjustments in a given aisle generate a peak of height $(n_{aisles})^{-1}$ in the firm-level daily share of price changes. If adjustment dates across isles do not coincide, one observes only peaks of size $(n_{aisles})^{-1}$. It is also possible, however, that adjustment days coincide for two or more aisles, in which case peak sizes are multiples of $(n_{aisles})^{-1}$. We refer to this model as the "aisle model."

Like the partial synchronization model, the aisle model also nests GL and ALM. When $n_{aisles} = n_{prod}$, products behave independently, and the model is equivalent to GL. If $n_{aisles} = 1$, the firm behaves as in the ALM model with a large number of products.

We calibrate $n_{prod} = 7200$, close to the sample average displayed in Table 1, and use the empirical FK statistic (0.258) to determine n_{asiles}. This yields $n_{aisles} = 15$, resulting in $n_{prod}/n_{aisle} = 480$ products per aisle. The remaining parameters—the menu cost and volatility of product-specific shocks—are calibrated to generate the same frequency of price changes and adjustment size as in the data.

By construction, the calibrated model matches the FK statistic in the data. Since it comprises a relatively small number of ALM economies with a large number of product aisles, it generates almost the same kurtosis of price changes as the ALM model (2.99 versus 3). Its ability to fit other pricing moments, however, is limited.

Figure 9 compares three sets of micro moments for the aisle and partial synchronization models: the daily fraction of price changes, and the distributions of price changes and price spells. The top left panel shows the daily share of adjustments. The main difference between the two models is that the aisle model generates variation in the intervals between peaks and in peak sizes. This pattern, however, is associated with full within-aisle synchronization. While peaks do vary in size and timing in the data, they tend to mix adjustments of products across the store, as in the partial synchronization model.[19] The top right panel shows that, unlike the partial synchronization model, the aisle model fails to generate the bimodality of the distribution of price changes, which highlights the important role of menu costs associated with each price change.

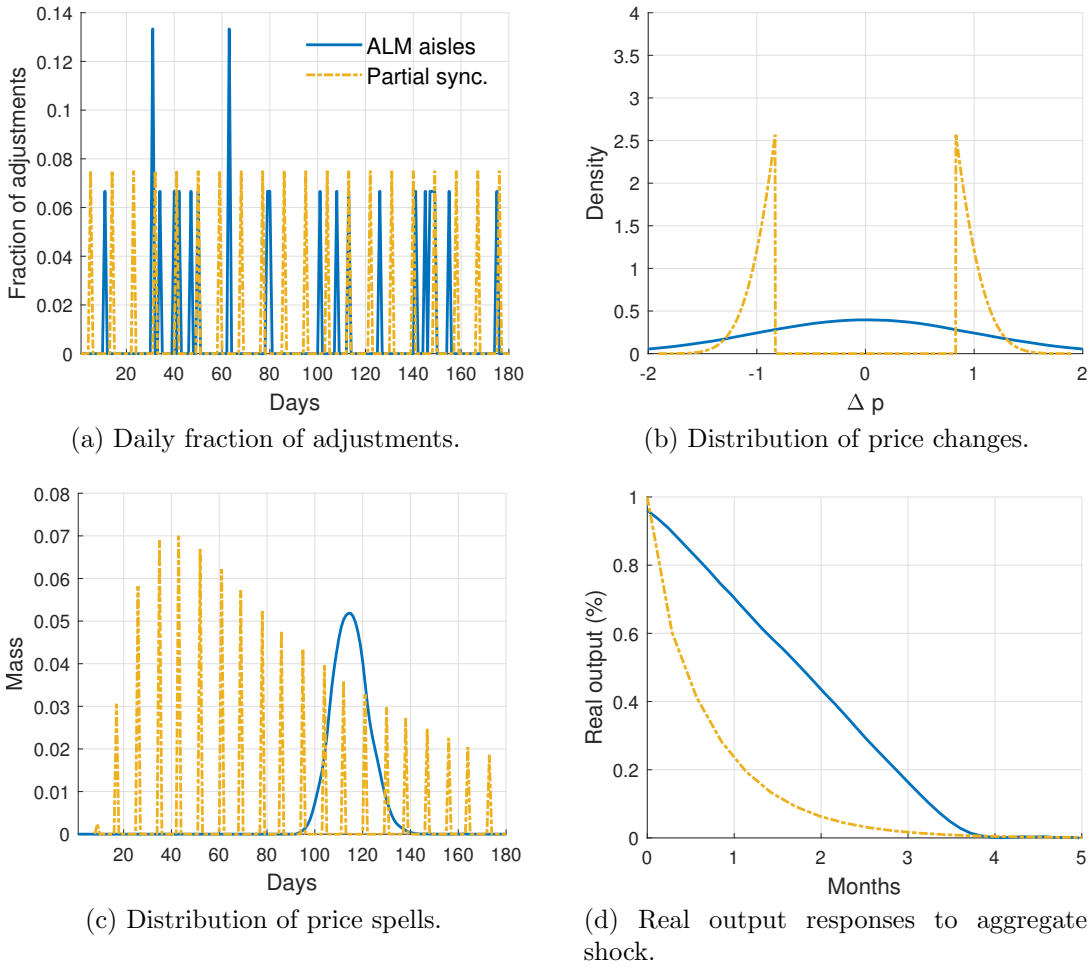
The bottom left panel shows the distribution of price spells. Despite featuring a finite number of products, the distribution implied by the aisle model is very concentrated,[20] with

[18] We thank a referee for encouraging us to analyze this alternative model.

[19] In Appendix A.4 we provide evidence that price adjustments on a typical peak day do not correspond to only a few product categories. When we condition the FK statistic on adjustments within the same product category, the increase in synchronization is small, from 0.248 to 0.285.

[20] A version of the aisle model with a continuum of products implies a degenerate distribution of price spells, as in ALM.

standard deviation of only 7.6 days—substantially smaller than 96 days observed in the data. The partial synchronization model, on the other hand, generates a much larger standard deviation of price spells: 85 days, which is close to the data.



(a) Daily fraction of adjustments.

(b) Distribution of price changes.

(c) Distribution of price spells.

(d) Real output responses to aggregate shock.

Figure 9: Micro moments for the aisle and partial synchronization models.

Finally, the bottom right panel shows the real output responses to nominal aggregate demand shocks. Not surprisingly, given its similarity to the standard ALM model, the aisle model generates substantially more monetary non-neutrality.

To summarize, the aisle model is an alternative way to generate partial synchronization in price adjustments. Due to the lack of individual menu costs, however, this model fails to produce a bimodal price change distribution, a salient feature of our data, even though it generates more realistic kurtosis. This suggests that the challenge posed by our data is to reconcile a relatively high kurtosis with the observed bimodality of price changes. Typical mechanisms for increasing kurtosis, such as free adjustment opportunities or perfect syn-

chronization in a multi-product setting, only do so by generating a large mass of small price changes. Empirically, this could be due to cross-sectional heterogeneity in pricing frictions (Klenow and Kryvtsov, 2008). Nevertheless, the partial synchronization model fares substantially better in matching the variability of price spells, an untargeted moment associated with a sufficient statistic in time-dependent pricing models (Carvalho and Schwartzman, 2015).

6 Conclusions

The literature on multiproduct pricing is still scarce and relatively new; there is still dearth of evidence and theory to help us understand behavior of large retailers and their impact on the economy. This paper contributes on both fronts. On the empirical end, we exploit unique data on day-to-day price changes across thousands of products sold in large food retailers in Israel. A typical retailer synchronizes its regular price changes around occasional peak days when a sizable share of store prices are simultaneously adjusted. This behavior suggests that retailers actively exploit substantial economies of scope in their day-to-day price adjustment. This finding is robust to incorporating price discounts, online prices, chain effects, and calendar effects.

On the theory side, we provide a generalization of existing models to a more flexible setting with economies of scope in price adjustment. Depending on the parameter configuration, our model can match an arbitrary degree of partial synchronization. When our model matches the degree of syncronization found in the data, it generates almost as much selection as in Golosov and Lucas (2007)'s model. Therefore, our results suggest synchronization of price adjustments by multiproduct firms is unlikely to be a relevant mechanism behind the high levels of kurtosis observed in the data. This gap can be addressed in future research, where developing settings with cross-sectional heterogeneity in pricing frictions and improving measurement of pricing for consumption products seems promising.

Because our dataset covers only food retailers, extrapolating our results to the whole economy requires some caution. In the paper, we find partial synchronization of price changes in online stores and in medium-size stores, and within both food and non-food product groups. Therefore, the evidence suggests our findings may apply to a broader population of retailers and products. Going forward, improved datasets can provide a more balanced representation of the economy by incorporating small- and medium-size stores, and by including services, durables and other goods.

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