Heterogeneous Agent New Keynesian Models: Day 1

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Rodolfo Rigato
European Central Bank

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Overview

- Two day course on Heterogeneous Agent New Keynesian (HANK) models
- Day 1: The canonical HANK model
  - Implications for monetary and fiscal policies
  - Introduce the notion of sequence space Jacobians
  - What micro moments matter for the transmission of macro shocks?
- Day 2: Advanced topics
  - Sequence Space Jacobian solution method
  - Matching the micro: Two asset models
  - Matching the macro: Inattention and other types of information frictions
  - An illustration using an estimated HANK model for the Euro Area

Some history: What took us so long?

- Seminal paper: Kaplan, Moll and Violante (2018), *"Monetary policy according to HANK"*
  - Heterogeneity drastically changes monetary transmission
- But both HA and NK models have been used for around 30 years now
  - Why did we wait so long to combine HA and NK into HANK?

1. Data: Early HA models were not designed to match the right micro moments
   - MPCs, household balance sheets, income and unemployment risk, etc.
2. Technology: Solving HA model with aggregate shocks is not trivial
   - The entire distribution of agents is a state variable
   - Efficient (and user-friendly) solution methods are fairly recent

The macroeconomic priorities of 20 years ago

- Robert Lucas's 2003 AEA presidential address titled "Macroeconomic Priorities"

These exogenous forces acting differentially across households induce different individual choices, which in turn lead to differences in individual capital holdings. The state space in this economy is very large, much larger than anything people were working with numerically 15 years ago, and without the method developed in Krusell and Smith (1998) it would not have been possible to work out the predictions of this model. [...] ***For determining the behavior of aggregates, they discovered, realistically modeled household heterogeneity just does not matter very much.***
For individual behavior and welfare, of course, heterogeneity is everything.

- As we will see now, recent research suggests it actually matters for aggregates too

The canonical HANK model

- Based on Auclert, Rognlie and Straub, 2023 and 2024 (henceforth ARS)
- Continuum of households indexed by i, with preferences given by

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t\left(\log c_{it}-\frac{n_{it}^{1+\chi}}{1+\chi}\right)$$

- Heterogeneity in labor skill e_{it} (assume $\mathbb{E}_i e_{it}=1$) and asset holdings b_{it}
- After tax labor income is $z_{it}=(1-\tau_t)w_t n_{it} e_{it}$
- Households save in riskless government bonds:

$$c_{it}+b_{it}=z_{it}+(1+r_{t-1})b_{i,t-1}$$

- Borrowing constraint: $b_{it}\geq \underline{b}$

The canonical HANK model: Supply side

- There is a representative firm that employs labor N_t and produces output $Y_t = N_t$
- Prices are flexible, so $P_t = W_t \implies w_t = 1$
  - Helpful assumption in HANK models because it implies no dividends
- Wages are sticky and decided by unions
  - Assume equal hours $n_{it} = N_t$
  - Maximize the welfare of an agent with average consumption C_t
  - This results in the follow Phillips curve:

$$\pi_t = \kappa \left(v'(N_t) - \frac{(1-\tau_t)w_t}{\mu} u'(C_t) \right) + \beta \pi_{t+1}$$

- No expectation over π_{t+1} since we will look at first order, perfect foresight shocks

The canonical HANK model: Government

- The fiscal authority raises taxes T_t to finance spending G_t and debt B_t

$$B_t = (1 + r_{t-1})B_{t-1} + G_t - T_t$$

- Total tax revenue is

$$\begin{aligned} T_t &= \tau_t w_t N_t \\ &= \tau_t Y_t \end{aligned}$$

- The monetary authority follows the rule:

$$1 + i_t = (1 + r)(1 + \pi_{t+1})^{\phi}$$

  - We consider the limit as $\phi \to 1$

- Whenever necessary, the fiscal authority follows the rule

$$\tau_t = \tau + \eta \frac{B_t - B}{Y}$$

The Intertemporal Keynesian Cross

- Start with fiscal policy: consider first-order fiscal shocks $\{dT_t, dG_t\}_{t=0}^{\infty}$ satisfying

$$\sum_{t=0}^{\infty} \frac{dT_t}{(1+r)^t} = \sum_{t=0}^{\infty} \frac{dG_t}{(1+r)^t}$$

- Household disposable income is

$$\begin{aligned} z_{it} &= (1-\tau_t) w_t n_{it} e_{it} \\ &= (Y_t - T_t) e_{it} \end{aligned}$$

  - Labor skill e_{it} determines each household's share of the pie

- **Aggregate consumption depends only on total disposable income** $Z_t = Y_t - T_t$
- Following an MIT shock, the goods market clearing condition is

$$Y_t = C_t\left(\{Z_s\}_{s=0}^{\infty}\right) + G_t$$

The Intertemporal Keynesian Cross

- **Proposition 1** (ARS, 2023) Linearizing the goods market clearing condition, we obtain:

$$d\mathbf{Y} = \mathbf{M} \cdot (d\mathbf{Y} - d\mathbf{T}) + d\mathbf{G}$$

- The matrix **M** has as entries the **intertemporal MPCs (iMPCs)**

$$M_{ts} = \frac{\partial C_t}{\partial Z_s}$$

  - It completely characterizes (dynamic) fiscal multipliers
  - It is a **sequence space Jacobian:** maps a sequence $\{Z_t\}_{t=0}^{\infty}$ to $\{C_t\}_{t=0}^{\infty}$
- Implication: balanced-budget policies ($dT_t = dG_t$) have unit multipliers:

$$d\mathbf{Y} = d\mathbf{G}$$

Representative agent

- Different models generate different iMPC profiles
  - Let's compare three popular models: RA, TA, and HA
- The representative agent model (with $\beta = 1/(1+r)$) has

$$\mathbf{M}^{RA} = \begin{pmatrix} (1-\beta) & (1-\beta)\beta & (1-\beta)\beta^2 & \cdots \\ (1-\beta) & (1-\beta)\beta & (1-\beta)\beta^2 & \cdots \\ (1-\beta) & (1-\beta)\beta & (1-\beta)\beta^2 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

- Some properties of $\mathbf{M}^{RA}$
  - MPCs are small because $\beta \approx 1$
  - Only the NPV of future income matters
  - Fiscal multipliers are always 1 (remember constant r)

Two agents

- A two-agent model is also a common specification
  - Many examples in central banks' quantitative DSGE models due to tractability
- A share μ of agents is liquidity constrained and consumes all its income
- The other $1 - \mu$ has access to complete markets
- What is the iMPC matrix?

$$\mathbf{M}^{TA} = \mu \cdot I + (1 - \mu)\mathbf{M}^{RA}$$

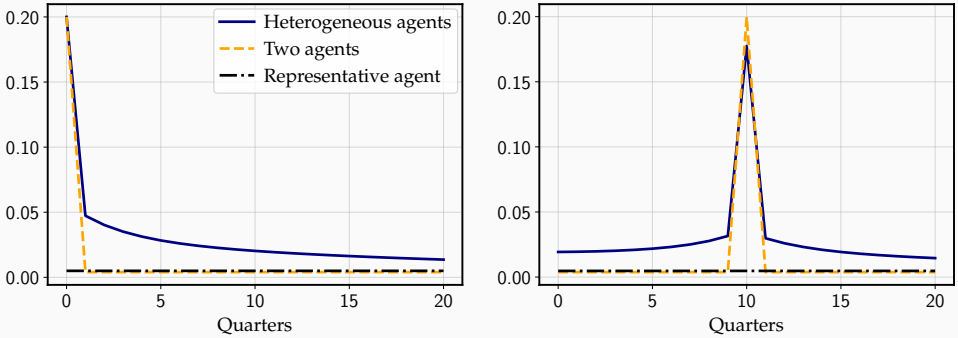
Heterogeneous agents

- With heterogeneous agents, no closed form solution in general
  - **M** needs to be computed numerically
- One special case worth mentioning: the zero liquidity limit (Werning 2015, Mankiw and Reis 2021, Bilbiie 2024):

$$\underline{b} = 0 \qquad B \to 0$$

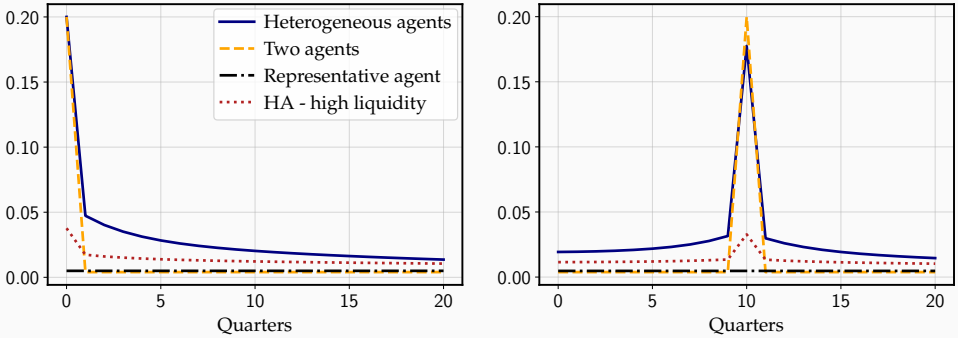
  - Tractable because there is no wealth distribution
  - Useful because the qualitative properties of more general HANK models still hold
  - More on this later

Comparing iMPCs



- Calibrated $M_{00} = 0.2$ (ARS 2024)
  - For the HA model, this requires $B/Y = 60\%$

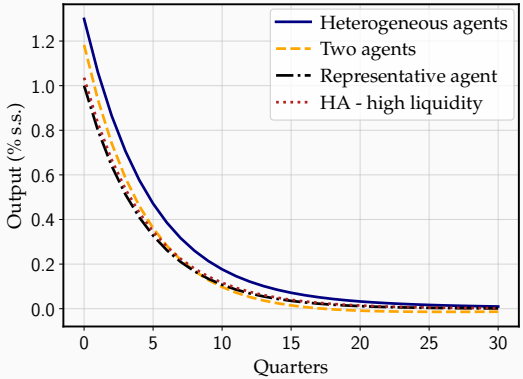
Comparing iMPCs



- Calibrated $M_{00} = 0.2$ (ARS 2024)
  - For the HA model, this requires $B/Y = 60\%$ – in the data, closer to $B/Y = 400\%$

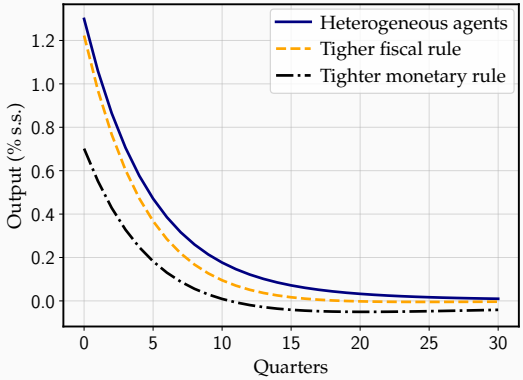
Fiscal multipliers

- Shock to government spending $dG_t = 0.8^t$



- Unit multipliers in with RA vs. 1.46 (cumulative) with HA
- RA and HA very similar with more liquid wealth: Krusell and Smith (1998)

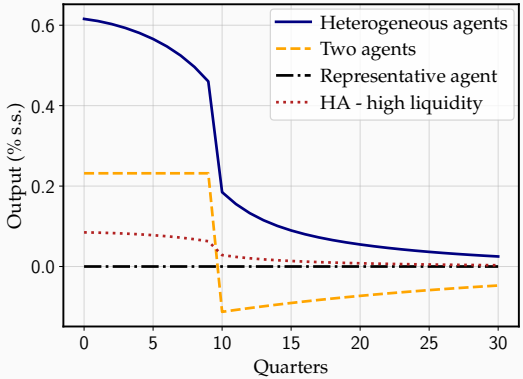
The importance of taxes and monetary policy



- Tighter fiscal rule: $\eta = 0.05 \rightarrow \eta = 0.15$
  - The degree to which the policy is debt financed affects multipliers
- Tighter monetary rule: $\phi = 1 \rightarrow \phi = 1.5$
  - Also known to be important in the representative agent model

Multipliers of tax cuts

- Taxes 1 p.p. below steady state for 10 quarters, then back to the rule



- No effect on representative agent model
- High multipliers in the HA model, also with consumption smoothing: **excess savings**

Monetary shocks

- Several assumptions were made so far to ensure that **M** is the only Jacobian
- With monetary shocks, the IKC becomes

$$Y_t = C_t\left(\{Z_s, r_s\}_{s=0}^{\infty}\right) + G_t$$

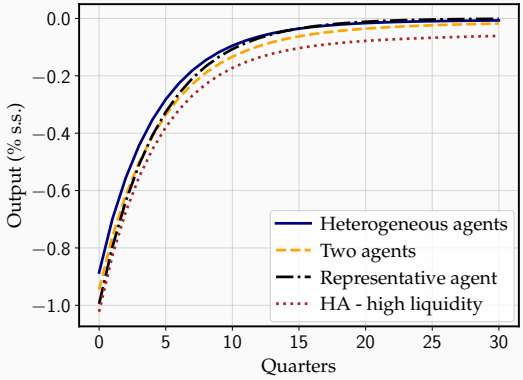
- Linearizing around the steady state

$$d\mathbf{Y} = \mathbf{M} \cdot (d\mathbf{Y} - d\mathbf{T}) + \mathbf{M}^r d\mathbf{r} + d\mathbf{G}$$

- This requires $\phi = 1$, otherwise need to incorporate the Taylor rule and Phillips curve

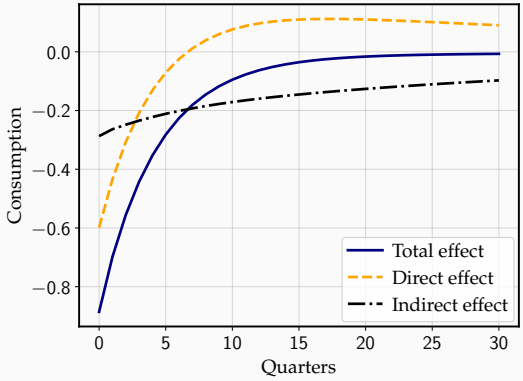
Monetary shocks

- Monetary shock has similar effects across models



Monetary shocks

- ...but different transmission channels in HA: Kaplan, Moll and Violante (2018)



$$d\mathbf{C} = \underbrace{\mathbf{M}^r d\mathbf{r}}_{\text{direct effect}} + \underbrace{\mathbf{M} \cdot (d\mathbf{Y} - d\mathbf{T})}_{\text{indirect effect}}$$

Exact result with zero liquidity

- Let's introduce now the **zero liquidity limit**: $\underline{b} = 0$ and $B \to 0$
- For simplicity, consider a two-state idiosyncratic process $\{e^L, e^H\}$
- Also for simplicity, symmetric transition matrix $\implies$ stationary distribution $(1/2, 1/2)$

$$\Pi = \begin{bmatrix} p & 1-p \\ 1-p & p \end{bmatrix}$$

- With zero liquidity,

$$c_t^H = (1-\tau_t) w_t N_t e^H \quad c_t^L = (1-\tau_t) w_t N_t e^L$$

- But the Euler equation of the high type agent still holds

$$u'(c_t^H) = \beta(1+r_t)\left[p u'(c_{t+1}^H) + (1-p) u'(c_{t+1}^L)\right]$$

Zero liquidity

- Assuming CRRA utility:

$$(1-\tau_t)w_tN_t = (1+r_t)^{-\frac{1}{\sigma}}\left\{\beta\left[p+(1-p)\left(\frac{e^L}{e^H}\right)^{-\sigma}\right]\right\}(1-\tau_{t+1})w_{t+1}N_{t+1}$$

- Market clearing:

$$Y_t - G_t = \frac{c_t^H + c_t^L}{2} = (1-\tau_t)w_tN_t$$

- We obtain the same Euler equation as in a RANK model

$$Y_t - G_t = \left(\frac{1+r_t}{1+r}\right)^{-\frac{1}{\sigma}}(Y_{t+1} - G_{t+1})$$

- But iMPCs are different! How is this possible?
  - More general result in ARS (2024)

Income risk and precautionary saving

- Bilbiie (2024): zero liquidity model with countercyclical risk (or inequality)
  - Gap between $\{e^L, e^H\}$ increases in recessions
  - Alternatively, countercyclical probability of moving from e^H to e^L (unemployment)
  - Empirical support for both
- Precautionary saving is another major force in HA models
- This generates amplification (or dampening) in the Euler equation

$$y_t = \lambda \mathbb{E}_t y_{t+1} - \frac{1}{\sigma}(i_t - \mathbb{E}_t \pi_{t+1})$$

  - With countercyclical risk, $\lambda > 1$
- Important implication: **the Taylor principle fails**
  - Determinacy requires $\phi > 1$

Other extensions: Open economy

- Auclert, Rognlie, Souchier and Straub (2021)
- Households consume domestic and imported goods:

$$C = \left[\alpha^{1/\eta} C_F^{(\eta-1)/\eta} + (1-\alpha)^{1/\eta} C_H^{(\eta-1)/\eta}\right]^{\frac{\eta}{\eta-1}}$$

- Following a shock to the real exchange rate Q:

$$d\mathbf{Y} = \left[\frac{\alpha}{1-\alpha}\chi - \alpha\mathbf{M}\right] d\mathbf{Q} + (1-\alpha)\mathbf{M}d\mathbf{Y}$$

- χ depends on the elasticity of substitution between domestic and imported goods
- An exchange rate depreciation can be **contractionary** if MPCs are high

Other extensions: Unemployment insurance

- Kekre (2023), Fernandes and Rigato (2024)
- Simple case: employed and unemployed agents

$$z_{it}^e = (1 - \tau_t) w_t N_t \qquad z_t^u = b$$

- An increase in UI generosity stimulates consumption through two channels:

1. **MPC heterogeneity:** tax the employed and give to the unemployed
2. **Precautionary saving:** reduces incentives of the **employed** to save

Takeaways

- Heterogeneity matters for the transmission of shocks and policies
- But you must match the right moments
- Everything related to MPCs is different in HANK:
  - Fiscal policy has higher multipliers
  - Monetary policy operates through different channels
  - An exchange rate depreciation can be contractionary
  - Opens the door to other widely used policies such as unemployment insurance
- Precautionary saving also plays an important role